ONE-DIMENSIONAL MASS AND HEAT TRANSFER AND THEIR COUPLING†

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Abstract—Consideration is given to the steady-state transfer of mass and heat from a porous wall to a non-dissipative binary system or air–hydrogen in one case, and air–carbon dioxide in another. Hydrogen or helium is injected at a uniform rate through the porous wall into the system. The flow, temperature, and concentration fields are assumed to be functions of the distance from the wall only. According to the thermodynamics of irreversible processes, the fluxes of heat and mass are coupled. The effects of such coupling are shown to be small on the concentration field, appreciable on the temperature field and heat flux under certain conditions, and negligible on the heat-transfer coefficient when defined in terms of an "adiabatic wall temperature". In general, the magnitude of the effects are much more pronounced for hydrogen injection than for carbon dioxide injection, and are in the opposite sense.

NOMENCLATURE

b, distance between wall and plane, Fig. 1;

c, mass fraction;

$\bar{c}_1$, average mass fraction of the injected gas between wall and plane;

C_p, specific heat at constant pressure;

D, coefficient of ordinary diffusion between injected gas and air;

G, mass flux crossing a fixed plane parallel to wall;

h, heat-transfer coefficient defined by equation (27);

J, mass flux crossing a plane parallel to wall and moving with mixture;

k, thermal conductivity;

$\dot{m}$, mass injection rate per unit area through wall;

M, molecular weight;

q, heat flux;

q^*, dimensionless heat flux $= (qb/kT)$;

R, gas constant;

T, temperature;

T_a, adiabatic wall temperature, temperature of plane for zero heat exchange between it and wall;

T^*, dimensionless temperature $= T/T_w$;

$T^{*\prime}$, dimensionless temperature gradient $= (\mathrm{d}T^*/\mathrm{d}y^*)$;

v, mixture velocity along the y-axis;

x, y, space co-ordinates, Fig. 1;

y^*, dimensionless distance from the wall $= y/b$.

Greek symbols

a, thermal diffusion factor;

a_0, reference thermal diffusion factor when $c_1 = 0$ in equation (19), or (21);

ρ, density.

Dimensionless moduli

$A = (\dot{m}b/\rho_2 D)$;

$Le = (k_2/\rho_2 C_{p2} D)$, Lewis number.

Subscripts

1, injected gas, hydrogen or carbon dioxide;

2, air;

w, wall;

b, plane.

1. INTRODUCTION

THE importance of the effects of the thermodynamic coupling between heat and mass transfer on heat transfer and adiabatic wall

temperature in binary systems have been published quite recently [1]. Measurements with helium injection into an incompressible turbulent boundary layer [2] clearly demonstrated experimentally significant effects of the coupling in an important engineering application of the injection process. These effects resulted in a rather unusual phenomenon in low speed flow. The wall temperature was found to be higher than the free stream temperature by up to about 40 degF for zero heat exchange with the boundary layer. Subsequent measurements [3], showed that the heat flux due to the coupling was of the same order of magnitude, and even exceeded, the familiar Fourier conduction heat flux, and was in the opposite direction. An analytical investigation [2] of the effects of helium injection on heat transfer in Couette flow taking the coupling into consideration in the conservation equations showed that the above phenomena were indeed the results of such coupling.

This paper investigates more completely and analytically the effects of the coupling when gases other than helium are mixed with air, and hence to obtain a better understanding of the parameters involved. Hydrogen and carbon dioxide are selected for this purpose because of their widely contrasting properties. The molecular weight of the first gas is about 0·07 that of air, and its thermal diffusion factor negative and one of the highest known. The molecular weight of the second gas is about 1·5 that of air and its thermal diffusion factor is positive and about one-fifth that of hydrogen. Moreover, hydrogen and carbon dioxide have substantially different thermodynamic properties pertinent to this analysis, such as specific heat and thermal conductivity. This investigation includes the effects of injection and of the coupling on gross quantities such as heat transfer at the wall and adiabatic wall temperature, as well as on the details of the concentration and temperature fields.

2. GOVERNING EQUATIONS

According to the principles of thermodynamics of irreversible processes, e.g. [4], the heat flux in a multicomponent system depends on the concentration fields of the different species in addition to its familiar dependence

on the temperature field. Similarly, the mass flux of a species depends on the temperature field in addition to its familiar dependence on the concentration field. In other words, the transfer of heat and mass are coupled. The general conservation equations taking such coupling into consideration were developed in detail in [5] for two-dimensional (x and y) flow, temperature, and concentration fields. They are simplified here when such fields depend on one space variable only (y), and when body forces, viscous dissipation and pressure diffusion are negligible.

This mathematical simplification describes approximately the physical system shown in Fig. 1, which lies between a porous wall and a hypothetical fixed plane parallel to it and lying in the boundary layer next to the wall. The hypothetical plane is assumed not to interfere with species transfer in the boundary layer by being sufficiently thin and porous or otherwise. The free stream is supposed to be air flowing at sufficiently low speeds to neglect viscous dissipation.

Hydrogen, or carbon dioxide, is injected through the porous wall into the boundary layer at a uniform rate per unit wall area. The free stream and porous wall are assumed at different temperatures. Under these conditions, changes in the x-direction are negligible in comparison with changes in the y-direction, so that all the dependent variables are approximately functions of the y-co-ordinate only.
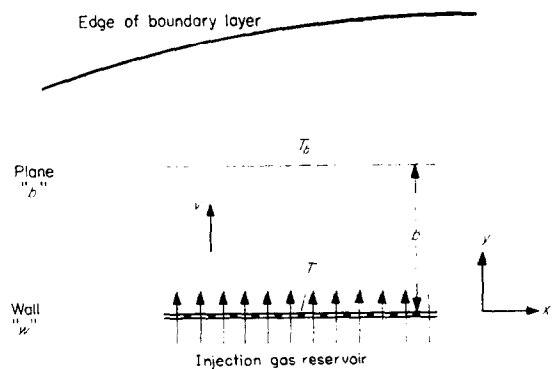

FIG. 1. One-dimensional heat- and mass-transfer system with foreign gas injection.

Conservation of mass of the mixture yields:

$$d(\rho v)/dy = 0. \qquad (1)$$

Conservation of mass of the injected species yields:

$$\rho v \,(dc_1/dy) = - \,(dJ_1/dy) \qquad (2)$$

where:

$$J_1 = - \,\rho D \,[(dc_1/dy) + c_1 c_2 \,a \,(d \ln T/dy)]. \qquad (3)$$

Conservation of energy of mixture yields:

$$\rho v \,C_p \,(dT/dy) = d\,[k \,(dT/dy)]/dy$$
$$- \,(aRTM^2/M_1 M_2) \,(dJ_1/dy)$$
$$- \,J_1 \,d\,[(C_{p1} - C_{p2}) \,T + aRTM^2/M_1 M_2]/dy. \qquad (4)$$

A. Solution of Equations (1) and (2)

The solution of equation (1) is easily determined to be:

$$\rho v = \text{constant} = \rho_w v_w. \qquad (5)$$

Substituting this result in equation (2) and solving it yields:

$$\rho_w v_w \,c_1 + J_1 = \text{constant}. \qquad (6)$$

Since the mass flux of the injected gas through any fixed plane parallel to the x-axis is given by:

$$G_1 = J_1 + \rho v \,c_1 \qquad (7)$$

it follows from equations (6) and (7) that

$$G_1 = \text{constant} = G_{1w} = \dot{m}. \qquad (8)$$

But the sum of the mass fluxes of the two species through a fixed plane is given by:

$$G_1 + G_2 = \rho v = \text{constant} = \rho_w v_w. \qquad (9)$$

Hence from equations (8) and (9), it follows that

$$G_2 = \text{constant} = G_{2w}.$$

In the following calculations it is assumed that no air passes through the wall and hence:

$$G_{2w} = 0. \qquad (10)$$

From equations (8), (9) and (10), it follows that:

$$\rho v = \rho_w v_w = G_1 = G_{1w} = \dot{m} = \text{constant}. \qquad (11)$$

From equations (6), (7) and (11), there results:

$$J_1 = \dot{m} \,(1 - c_1). \qquad (12)$$

B. Transformation of Equations (3) and (4)

Comparison of equations (3) and (12) yields:

$$\dot{m} \,(1 - c_1) = - \,\rho D \,\{(dc_1/dy)$$
$$+ c_1 c_2 \,a \,[d(\ln T)/dy]\}. \qquad (13)$$

Equations (4) and (13) are transformed into dimensionless form by introducing the following substitutions:

$$y^* = (y/b), \quad T^* = (T/T_w).$$

After some reduction, the transformed equations are respectively:

$$(C_{p1}/C_{p2}) \,(dT^*/dy^*)$$
$$= (Le/A) \,d \,[(k/k_2) \,(dT^*/dy^*)]/dy^*$$
$$- a_0 \,(R_2/C_{p2}) \,d\,[(a/a_0) \,(M/M_1) \,(1 - c_1) \,T^*]/dy^* \qquad (14)$$

and

$$(dc_1/dy^*) = - \,A \,(1 - c_1) \,(\rho_2/\rho)$$
$$- a_0 \,(a/a_0) \,c_1 c_2 \,d \,(\ln T^*)/dy^* \qquad (15)$$

in which A and Le are dimensionless groups given respectively by:

$$A = (\dot{m} \,b/\rho_2 D), \quad Le = (k_2/\rho_2 \,C_{p2} \,D)$$

which is the familiar Lewis number based upon air properties.

By inspection of equations (14) and (15), their solutions—and hence the heat and mass transfer—depend upon the parameters A, Le, (C_{p1}/C_{p2}), (R_2/C_{p2}) and a_0; as well as the ratios of mixture specific heat, thermal conductivity, molecular weight, and density to their respective values for air. These ratios depend upon the mass fraction of the injected gas, and the ratio of molecular weight of the injected gas to that of air.

All the parameters except A are combinations of thermodynamic properties of air and the injected gas, and hence will be prescribed when the injected gas is prescribed. The parameter A on the other hand contains the injection rate $\dot{m}$, and so is a measure of the injection rate when the distance between wall and plane, and the injected gas, are prescribed.

From kinetic theory, a_0 depends on the exponent in the force law during a molecular encounter, the mass of a molecule of injected

gas relative to that of air, as well as the diameter ratio [6]. The latter ratio is proportional to the former in most cases, and hence all the thermodynamic properties depend on the ratio of molecular weights, which therefore must be the parameter of greatest importance—besides A—affecting the solution of the equations.

C. *Boundary Conditions*

Two boundary conditions have already been specified in order to obtain complete solutions of equations (1) and (2), namely that at the wall $(\rho_w v_w)$ is specified, as well as $G_{1w} = \dot{m}$. Moreover, the assumption that $G_{2w} = 0$ was introduced in order to tie the quantity $(\rho_w v_w)$ with a measurable quantity, resulting in $\rho_w v_w = \dot{m}$.

Equations (3) and (4) were transformed into equations (15) and (14) respectively. Equation (14) requires two boundary conditions, and these were conveniently chosen as the temperature and the temperature gradient at the wall. Equation (15) requires one boundary condition, and this was chosen to be the mass fraction of the injected gas at the wall. Symbolically, these boundary conditions are expressed by the following relations respectively:

$$T_w^* = 1 \tag{16}$$

$$(dT^*/dy^*)_w = \text{an arbitrary number} \tag{17}$$

$$c_{1w} = \text{an arbitrary number.} \tag{18}$$

D. *Properties*

The temperature difference between the wall and plate was assumed sufficiently small to neglect property variations with respect to temperature.

The thermal diffusion factor α is a weak function of c_1. Figure 2(a) shows the results of measurements of α at various levels of concentration of hydrogen in mixtures with nitrogen [6]. For the purposes of the present analysis, a mean relationship was adopted for hydrogen-air mixtures according to the following relation:

$$\alpha = -0.2\,(1 + 8.5\,c_1) \text{ when } 0 < c_1 \leqslant 0.1, \tag{19}$$

$$\text{and} \quad \alpha = -0.37 \text{ when } 0.1 < c_1 < 1. \tag{20}$$

Various measurements of the thermal diffusion factor for CO_2-N_2 and CO_2-O_2 mixtures [6]
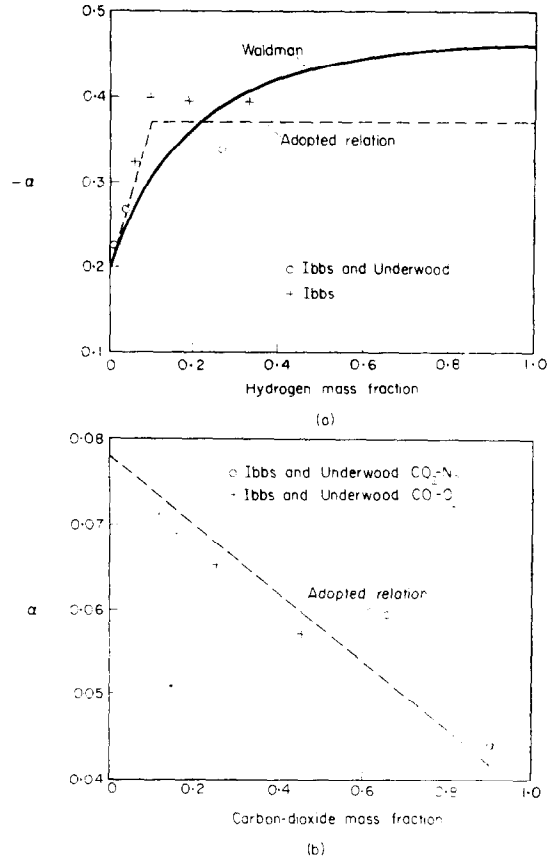

FIG. 2(a) and (b). Thermal diffusion factor.

are shown in Fig. 2(b). In the present analysis, the following relation was adopted for CO_2-air mixtures:

$$\alpha = 0.078\,(1 - 0.5\,c_1), \quad 0 < c_1 < 0.8. \tag{21}$$

At $500°R$, the following quantities have the indicated values

	H_2-Air	CO_2-Air
C_{p1}/C_{p2}	14.2	0.81
Le	0.3	1.2
$(M_2 a_0 R_2/C_{p2})$	-1.67	0.65

Regarding the ratio (k/k_2), Fig. 2 in [7] for hydrogen-air mixtures at $500°R$ was closely fitted with the following relationship:

$$(k/k_2) = 6.57 - 2.43\,(1 - c)^3 - 3.14\,(1 - c)^9.$$

The graph in [8] was closely fitted with the following relationship for carbon dioxide–air mixtures at 285°K:

$$(k/k_2) = 1 - 10.44\, c_1/(44 - 15\, c_1).$$

3. RESULTS AND DISCUSSION

A. Hydrogen Injection

After appropriate substitutions and re-arrangement, equations (14) and (15) become respectively:

$$[6.57 - 2.43\,(1 - c_1)^3 - 3.14\,(1 - c_1)^9]$$
$$(d^2T^*/dy^{*2})$$

$$= A\,[14.2 - 1.67\,(1 - c_1)$$
$$(1 + 8.5\,c_1)/(2 + 27\,c_1)]\,(dT^*/dy^*)/0.3$$
$$+ \{1.67\,A\,(1 - c_1)\,(1 + 8.5\,c_1)\,T^*\,[1/(1 - c_1)$$
$$+ 27/(2 + 27\,c_1) - 8.5P/(1 + 8.5\,c_1)]/0.3$$
$$(2 + 27\,c_1)$$
$$- [7.87\,(1 - c_1)^2 + 28.6\,(1 - c_1)^8]$$
$$(dT^*/dy^*)\}\,(dc_1/dy^*) \quad (22)$$

and

$$(dc_1/dy^*) = - A\,(1 - c_1)\,(1 + 13.5\,c_1) +$$
$$+ 0.2\,c_1\,(1 - c_1)(1 + 8.5\,c_1)\,(dT^*/dy^*)/T^*. \quad (23)$$

In equations (22) and (23), the bracket $(1 + 8.5\,c_1)$ was replaced with 1.85 and P with zero whenever c_1 exceeded 0.1.

The two non-linear coupled differential equations (22) and (23) were solved on the Univac 1103. The parameter A and c_{1w} were arbitrarily chosen, and profiles of T^* and c_1 computed when $(dT^*/dy^*)_w$ was given several arbitrary values. The objective was to obtain solutions of the equations at various temperature levels of the plane relative to the wall, but with everything else unchanged.

Then c_{1w} was assigned several other arbitrary values while the parameter A was kept unchanged, and the above process repeated. The objective was to investigate the effects of changing the hydrogen concentration level at the wall.

Finally, A was assigned a different value, and all the processes above repeated, the objective being to investigate the effects of changing A.

From all these calculations, it was possible to determine the effects of the thermodynamic coupling between heat and mass transfer on the temperature and concentration fields, and on the heat transfer. The results are discussed in some detail below.

1. Concentration and temperature fields

Selected results are shown in Fig. 3 when $(T^{*\prime})_w = 0.1$ and -0.1, meaning that the plane is warmer and colder than the wall respectively. When the coupling is neglected by putting $a_0 = 0$ in equations (14) and (15), the concentration field is shown by the dotted line, and becomes independent of the value of $(T^{*\prime})_w$. It is well known that in a mixture of hydrogen and air, the former will spontaneously migrate to warmer regions. This behaviour explains the pattern of Fig. 3, in which the concentration at the plane is larger than that when the coupling is neglected if it is warmer than the wall, and smaller if it is colder than the wall. Therefore, the temperature field influences the concentration field. When $(T^{*\prime})_w = \pm 0.1$, and $c_{1w} = 0.04$, the percentage change in mass fraction at the plane due to the coupling amounts to about ± 4 per cent of the mass fraction when the coupling is neglected, and is thus small. This percentage
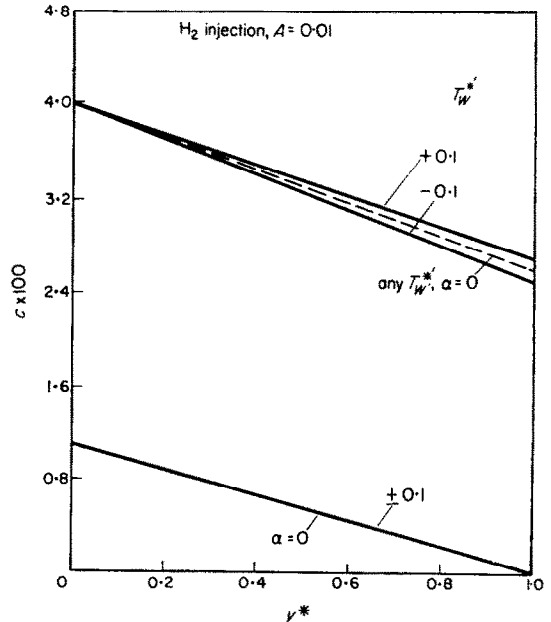

FIG. 3. Effect of the coupling on the hydrogen concentration field, for various concentration levels and temperature gradients at wall.

change becomes even smaller when the temperature difference between the plane and wall is reduced by reducing $(T^{*\prime})_w$.

The above procedure was repeated when $c_{1w} = 0{\cdot}03, 0{\cdot}02$, and $0{\cdot}011$. If c_{1w} is appreciably reduced below $0{\cdot}011$, negative values of c_{1b} will result. Since a negative mass fraction has no physical significance, it appears that c_{1w} has a minimum value of about $0{\cdot}011$. In this special case, the concentration field is also indicated in Fig. 3. Alternatively, negative values of c_1 in the field could mean that c_1 should be zero when $\delta^* \leqslant y^* \leqslant 1$ which physically means that the boundary-layer thickness δ^* is less than the distance between wall and plane. Such case will not be discussed in this analysis, since it is not compatible with the one-dimensional approximation postulated here.

The percentage change in c_{1b} due to the coupling is quite large when $c_{1w} = 0{\cdot}011$, being about ± 40 per cent when $(T^{*\prime})_w$ is $\pm 0{\cdot}1$. This percentage change decreases rapidly, however, to about 5 per cent, when $c_{1w} = 0{\cdot}02$ or $0{\cdot}03$.

The concentration field was also computed for the following cases:

$A = 0{\cdot}1$; $c_{1w} = 0{\cdot}2, 0{\cdot}4, 0{\cdot}6, 0{\cdot}7, 0{\cdot}8$;

$(T^{*\prime})_w$ varying from $-0{\cdot}05$ to $0{\cdot}05$.

$A = 0{\cdot}25$; $c_{1w} = 0{\cdot}9, 0{\cdot}95, 0{\cdot}97, 0{\cdot}99$.

$(T^{*\prime})_w$ varying from $-0{\cdot}05$ to $+0{\cdot}05$.

The same pattern observed for $A = 0{\cdot}01$ was also observed for $A = 0{\cdot}1$ and $0{\cdot}25$. In particular, the percentage change in c_{1b} due to the coupling was largest when c_{1w} was smallest.

To explain this observation, let us introduce the quantity defined by $(c_1 - \bar{c}_1)/\bar{c}_1$. It represents the deviation of the mass fraction of the injected gas at a particular location in the field from the field from the average mass fraction in the field. By inspection of Fig. 3, when c_{1w} is smallest, $(c_1 - \bar{c}_1)/\bar{c}_1$ is largest at plate and wall, and hence the departure from uniformity in con-

centration in the field is largest. Consequently, for a given value of A and T_b^*, the effects of the coupling on concentration are most pronounced when the distribution of mass fraction in the channel deviates the most from the uniform state.

The temperature fields are shown in Figs. 4(a) (b) and (c). When $A = 0{\cdot}01$, the coupling has negligible effects on the temperature field, Fig. 4(a). When $A = 0{\cdot}25$, however, its effects are most pronounced for $c_{1w} = 0{\cdot}8$, Fig. 4(b), which is the smallest wall mass fraction attempted, but are negligibly small for $c_{1w} = 0{\cdot}99$, Fig. 4(c). Similarly, when $A = 0{\cdot}1$, the effects are most pronounced at the smallest value of c_{1w} of $0{\cdot}2$, and negligible at the largest value of c_{1w}. Hence, the effects of the coupling on the temperature field are negligible for sufficiently small values of A, but can be large for higher values of A. In this latter case, they are most pronounced when the mass fraction distribution deviates the most from the uniform state.

2. Heat transfer

According to the thermodynamics of irreversible processes, e.g. [4, 5], the heat flux at wall I may be defined by:

$$q_w = [-k\,(\mathrm{d}T/\mathrm{d}y) + a\rho v RT M^2$$
$$(1 - c_1)/M_1 M_2]_w. \quad (24)$$

The first term on the right-hand side is the familiar heat transfer by conduction according to Fourier's Law, and the second term is contributed by the coupling between heat and mass transfer. This second term will vanish if $a = 0$ which means neglecting the coupling altogether, or if $(\rho v)_w = 0$, which is equivalent to no gas injection at the wall.

By transforming into dimensionless form and appropriate substitutions for property relations, equation (24) becomes

$$q_w^* = (q\,b/k\,T)_w = -\left\{\frac{\mathrm{d}T^*}{\mathrm{d}y^*} + A\frac{5\,(1 - c_1)\,(1 + 8{\cdot}5\,c_1)}{0{\cdot}9\,(2 + 27\,c_1)\,[6{\cdot}57 - 2{\cdot}43\,(1 - c_1)^3 - 3{\cdot}14\,(1 - c_1)^9]}\right\}_w,$$

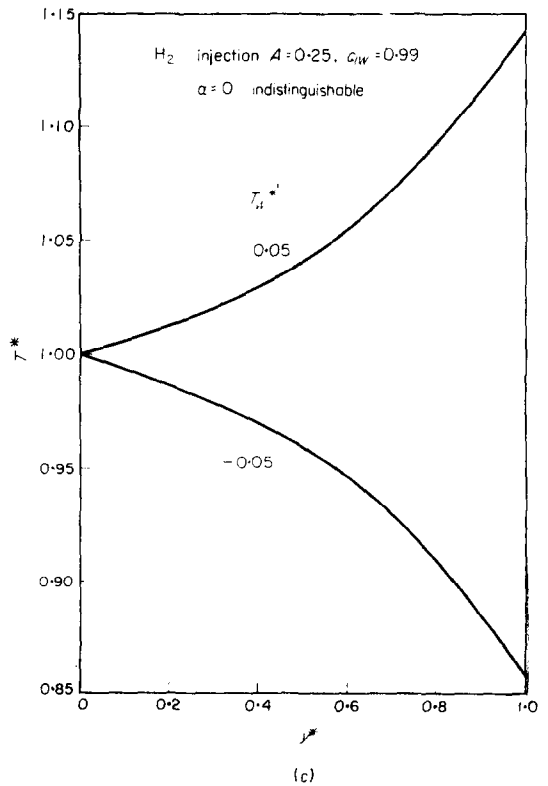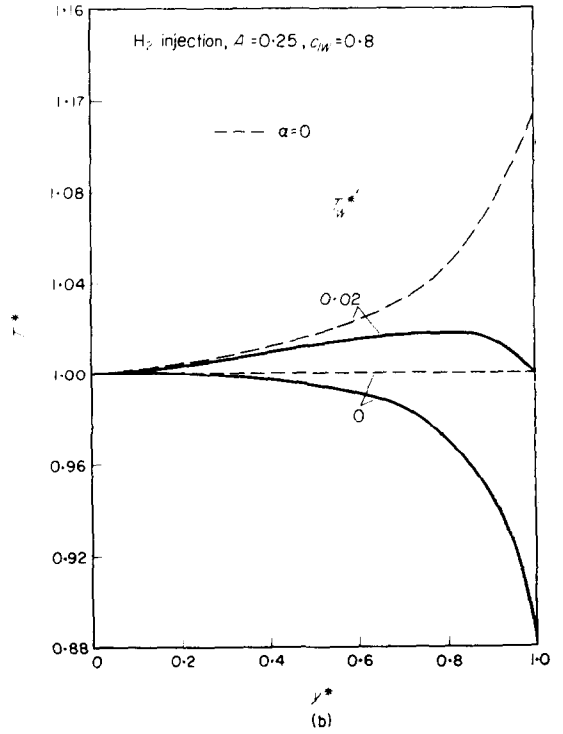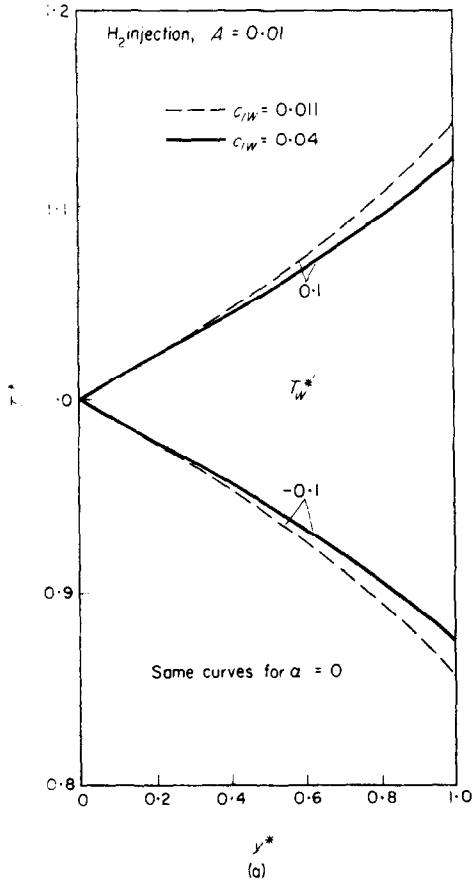$$\text{when } c_{1w} < 0{\cdot}1 \quad (25)$$

FIG. (4a), (b) and (c). Effect of the coupling on the temperature field for various values of hydrogen concentration and temperature gradient at wall, and A.

and:

$$q_w^* = -\left\{\frac{dT^*}{dy^*} + A \frac{9 \cdot 25 \, (1 - c_1)}{0 \cdot 9 \, (2 + 27 \, c_1) \, [6 \cdot 57 - 2 \cdot 43 \, (1 - c_1)^3 - 3 \cdot 14 \, (1 - c_1)^9]}\right\}_w, \quad \text{when } c_{1w} > 0 \cdot 1. \quad (26)$$

From the results of the calculations, q_w^* was computed and selected results plotted versus T_b^* in Fig. 5 for $A = 0 \cdot 25$. The graph is similar to Fig. 15 [2], after suitable modification of abscissa of the latter. Two conclusions can be immediately seen. First, q_w^* is a linear function of T_b^*, and hence is a linear function of the temperature difference between the wall and plane. This conclusion is similar to heat transfer in single component systems. Second, q_w^* does not vanish when the wall and plane have the same temperature, i.e. when $T_b^* = 1$. With hydrogen injection, q_w^* vanishes when T_b^* is less than 1, that is when the plane is colder than the wall. This conclusion is quite different from single component systems. It must be due to the coupling between heat and mass transfer, because when α is set equal to zero, q_w^* vanishes when $T_b^* = 1$ exactly. Relations similar to Fig. 5 were obtained when $A = 0 \cdot 1$ and $0 \cdot 01$. The effects of the coupling, however, were seen to diminish as A decreased.

The temperature of the plane at which the wall heat flux vanishes will be called the adiabatic wall temperature T_a. By setting $q_w^* = 0$ in equation (25) or (26), it was possible to compute T_a^* for various values of c_{1w} with A as parameter. The results are plotted in Fig. 6, along with the adiabatic wall temperature for helium injection [2] for comparison. It is seen that for a given value of A, T_a^* is least when c_{1w} is least, i.e. when the concentration field deviates the most from uniformity. For a given value of c_{1w}, T_a^* is least when A is greatest, i.e. when the injection rate is greatest.

From Fig. 6, an estimate of T_a can be made. If T_w is 500°R, then T_a will be less than T_w by up to about 10°F for $A = 0 \cdot 01$, and up to about 100°F when $A = 0 \cdot 1$ or $0 \cdot 25$, depending in each case upon the hydrogen mass fraction at the wall.

The linear relationships between q_w^* and T_b^* and the existence of an "adiabatic wall temperature" suggest the appropriate definition of the heat-transfer coefficient to be

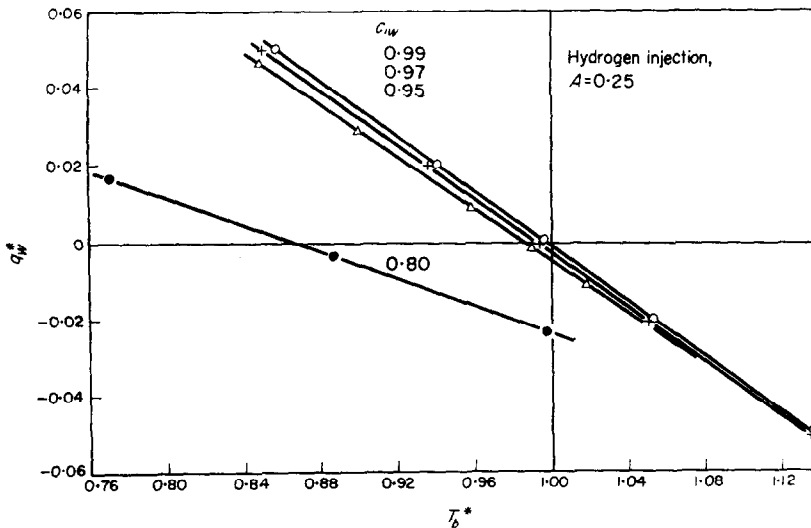$$q_w = h(T_a - T_b). \quad (27)$$

FIG. 5. Dependence of the heat flux on the temperature difference between wall and plane.
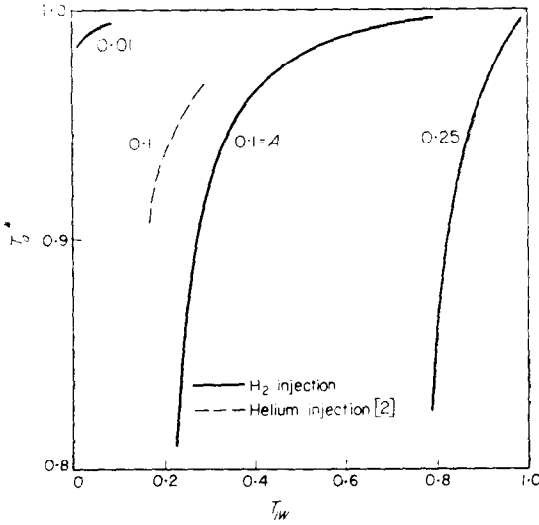
FIG. 6. Dependence of the adiabatic wall temperature on the hydrogen concentration level at wall for various values of A.
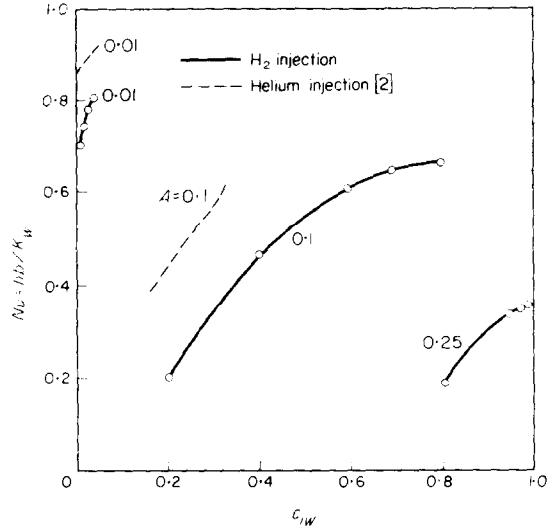

FIG. 7. Effect of hydrogen injection rate and concentration at wall on the Nusselt number.

In dimensionless form, equation (27) becomes:

$$q_w^* = (bq_w/T_w k_w) = (hb/k_w)(T_a^* - T_b^*)$$
$$= Nu_*^*(T_a^* - T_b^*). \quad (28)$$

From equation (28), the Nusselt number (hb/k_w) is simply the slope of the various straight lines in Fig. 5 and the similar figures. On comparing it with the Nusselt number when the coupling is neglected, the difference is about 3 per cent or less, and hence negligible. The Nusselt number is presented in Fig. 7, along with the Nusselt number for helium injection [2] for comparison.

B. Carbon Dioxide Injection

Equations (14) and (15) in this case become respectively:

$$(1 - 0.36 c_1)(d_2 T^*/dy^{*2})$$
$$= A [0.81 + 0.65 (1 - c_1)$$
$$(1 - 0.5 c_1)/(44 - 15 c_1)] (dT^*/dy^*)/1.2$$
$$- \{0.65 A T^*(1 - c_1)(1 - 0.5 c_1)$$
$$[1/(1 - c_1) + 0.5/(1 - 0.5 c_1)$$
$$- 15/(44 - 15 c_1)]/1.2 (44 - 15 c_1)$$
$$- 459 (dT^*/dy^*)/(44 - 15 c_1)^2\} (dc_1/dy^*)$$
$$(29)$$

$$(dc_1/dy^*) = - A (1 - c_1)(1 - 0.341 c_1)$$
$$- 0.078 (1 - 0.5 c_1) c_1 (dT^*/dy^*)/T^*. \quad (30)$$

Solutions were obtained in the same fashion as for hydrogen injection.

The concentration distribution is presented in Fig. 8 for the highest value of A attempted. When α is set equal to zero, the results are indistinguishable, showing that the coupling has negligible effects on the concentration field. Similar conclusions were observed for smaller values of A.

The temperature field is presented in Fig. 9, for the highest value of A. It is seen that for a given temperature gradient at the wall, the coupling has noticeable effects on the temperature field. It increases the temperature above its value for $\alpha = 0$, the increase being maximum at the plane, and mounting to about 1 per cent. This effect is opposite to that for hydrogen injection shown in Fig. 4. When $c_{1w} = 0.99$, the corresponding curves for $\alpha = 0$ are indistinguishable from the curves including α. Thus the effects of the coupling are most pronounced when c_{1w} is least, i.e. when the deviation of the concentration field from uniformity is greatest.

When $A = 1$, similar trends to the above are observed. When $A = 0.1$ however, no effects of
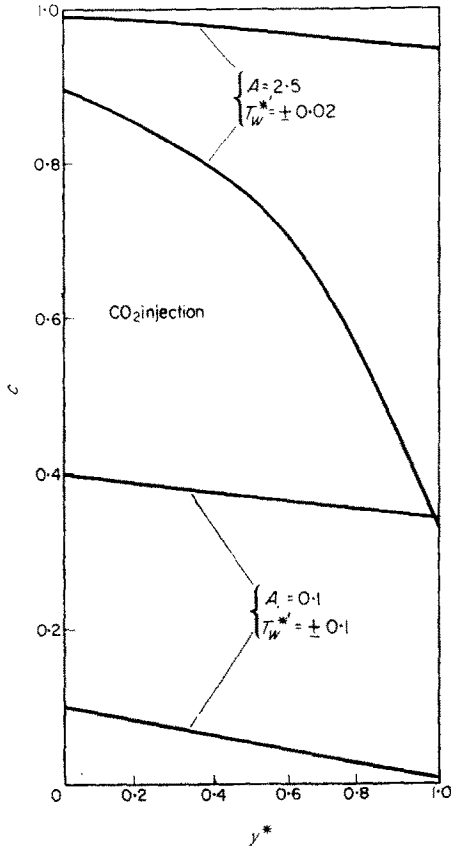
FIG. 8. Effect of the coupling on the carbon dioxide concentration field, at various values of concentration and temperature gradients at wall, and A.
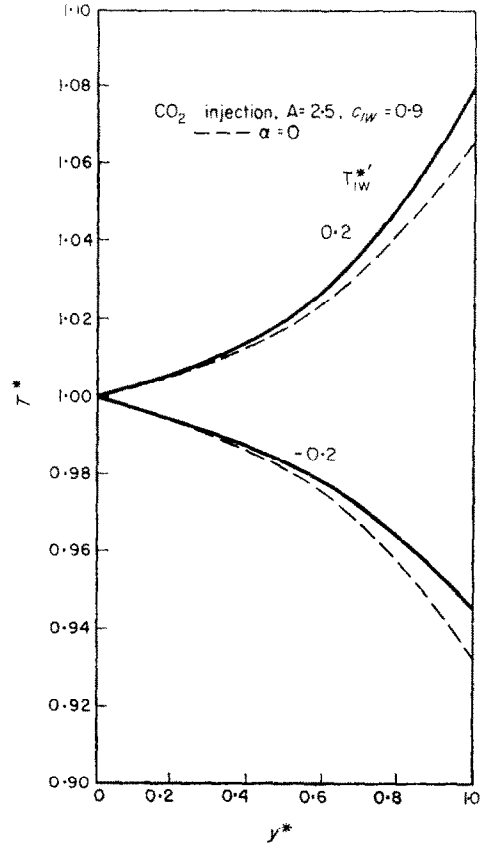

FIG. 9. Effect of the coupling on the temperature field.

the coupling on the temperature field could be seen.

Equation (24) is transformed for carbon dioxide injection into the following expression for the dimensionless heat flux at the wall:

$$q_w^* = \frac{q_w b}{k_w T_w} = -\left(\frac{dT^*}{dy^*}\right)_w$$

$$- A \frac{0.65(1 - 0.5 c_{1w})(1 - c_{1w})}{1.2(44 - 25.44 c_{1w})}. \quad (31)$$

Selected results are presented in Fig. 10 which is similar to Fig. 5 except that T_a^* is now greater than 1. This means that with carbon dioxide injection, the heat flux vanishes when the plane is warmer than the wall. From Fig. 10, it is seen that the excess of adiabatic wall temperature

over the wall temperature is maximum when c_{1w} is least, corresponding to the case of maximum deviation from uniformity of the concentration field between the two plates.

To investigate more fully the behaviour of T_a, equation (31) was solved subject to the condition that $q_w^* = 0$. The results are presented in Fig. 11. Again the heat-transfer coefficient should be defined by equation (27), and in dimensionless form is simply the slope of the straight lines in Fig. 10. When compared with its value when $\alpha = 0$, no effects of the coupling could be detected. The results are presented in Fig. 12.

4. SUMMARY AND CONCLUSIONS

Consideration is given to one-dimensional mass transfer and heat transfer and their coupling with the injection at a uniform rate of hydrogen in one case, and carbon dioxide in
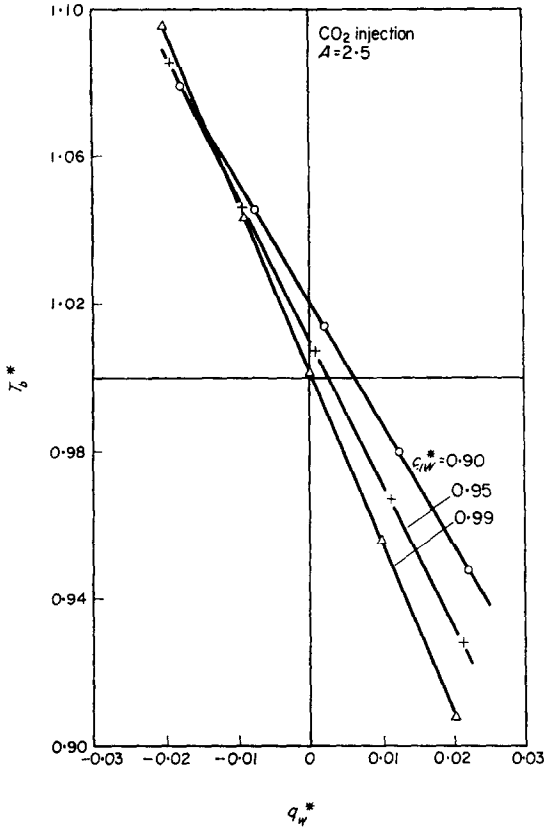
FIG. 10. Dependence of the heat flux on the temperature difference between wall and plane at various carbon dioxide concentration levels at wall.

FIG. 11. Dependence of the adiabatic wall temperature on the carbon dioxide concentration level at wall, for various values of A.

another case, through a wall into the adjacent binary flow field. These two particular gases were selected because of their widely contrasting properties that are pertinent to the present investigation. Compared to air, hydrogen density is about one-fifteenth, its thermal conductivity about seven times, and its specific heat fourteen times the corresponding properties of air. When mixed with air, it will migrate spontaneously to warmer regions. On the other hand, carbon dioxide density is one and a half times, its thermal conductivity about two-thirds, and its specific heat about 0·85 the corresponding properties of air. When mixed with air, it will migrate spontaneously to colder regions, and its thermal diffusion factor in air is about one-fifth that of hydrogen in air.

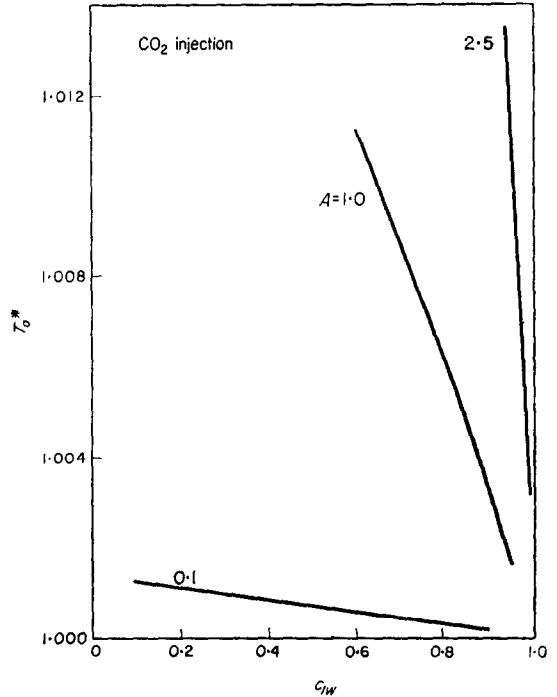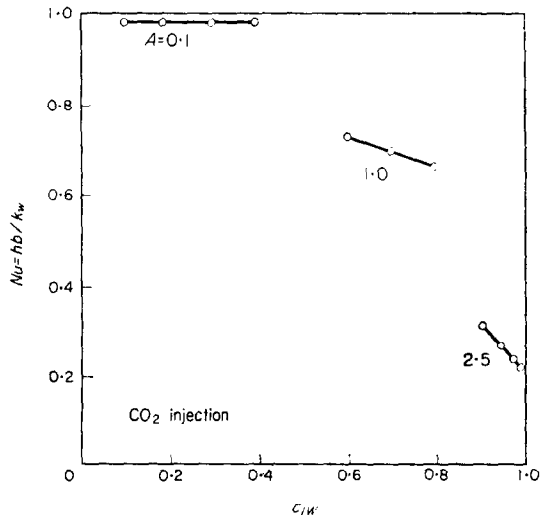In forming the governing equations, all

FIG. 12. Effect of carbon dioxide injection rate and concentration at wall on the Nusselt number.

property values were considered independent of temperature, but functions of the mass fraction of hydrogen or carbon dioxide when considering their mixtures with air. Solutions of the diffusion equation and the energy equation for a given injected gas depended upon the parameter A and the concentration level of the injected gas at the wall. They were obtained numerically on the Univac 1103, and compared with the solutions when the thermal diffusion factor was set equal to zero. Certain effects of the coupling between heat and mass transfer could thus be determined. In general, these effects were much more pronounced for hydrogen injection than for carbon dioxide injection, due to the ratio of molecular weight of hydrogen to air being much smaller than of air to carbon dioxide.

Due to the coupling, the hydrogen concentration at a particular point is slightly increased when the plane is warmer than the wall, and slightly decreased when the plane is colder than the wall. The opposite takes place with carbon dioxide injection, because of the opposite direction of migration of hydrogen and carbon dioxide in their respective non-isothermal mixtures with air.

The coupling has much more pronounced effects on temperature distribution than on concentration distribution, provided that A is sufficiently high, and that the deviation of mass fraction distribution from uniformity is sufficiently large. This deviation is largest when the mass fraction of the injected gas at the wall is the smallest possible compatible with non-negative mass fraction at the plane. Due to the coupling, the temperature at a particular point is decreased with hydrogen injection, and increased with carbon dioxide injection.

The wall heat flux is the sum of two components. The first component is the familiar Fourier conduction heat flux, and varies linearly as usual in single component systems with the temperature difference between the wall and the plane, for a given value of A and c_{1w}. The second component is caused by the coupling, is constant for a given value of A and c_{1w}, and in particular is independent of the temperature difference between the wall and plane. Its direction is opposite to that of the hydrogen mass flow through the wall, but the same as that

of the carbon dioxide mass flow. For a given value of A, this component is maximum when the concentration distribution deviates the most from uniformity.

Due to the coupling, the wall heat flux vanishes when the plane is colder than the wall with hydrogen injection, and warmer than the wall with carbon dioxide injection. When this happens, the temperature of the plane is called the adiabatic wall temperature. For a given value of A, the difference between adiabatic wall temperature and the wall temperature increases as the deviation of the concentration field from uniformity increases. Also, it becomes more pronounced when the injection is increased by increasing A.

The heat-transfer coefficient should be appropriately defined by equation (27) in terms of the adiabatic wall temperature. With this definition, its value becomes independent of the coupling.

In general therefore, the effects of the coupling are most pronounced when the distribution of mass fraction of either component, or the temperature distribution, deviates the most from the uniform distribution. They also increase as the ratio of molecular weights of the two species in the binary system, or the parameter A increases.

ACKNOWLEDGEMENTS

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Résumé—On s'est occupé du transfert de masse et de chaleur en régime permanent à partir d'une paroi poreuse vers un système binaire "non dissipatif" d'air et d'hélium dans un cas et d'air et de CO² dans l'autre cas. On ajoute dans le système de l'hélium ou de l'hydrogène à un débit uniforme à travers la paroi poreuse. On a suppose que les champs d'écoulement de température et de concentration sont seulement fonction de la distance à la paroi. Les flux de chaleur et de masse sont couplés en accord à la thermodynamique des processus irréversibles. On a montré que les effets d'un tel couplage sur le champ des concentrations sont faibles, sont appréciables sur le champ des températures et le flux de chaleur sous certains conditions, et négligeables sur le coefficient de transfert de chaleur lorsqu'il est défini à l'aide d'une "température de paroi adiabatique". En général, les effets sont beaucoup plus prononcés en valeur absolue pour l'injection d'hydrogène que pour celle de gaz carbonique et sont en sens opposé.

Zusammenfassung—Der stationäre Wärme- und Stoffübergang von einer porösen Wand an ein nicht-dissipatives binäres System, oder ein Luft-Wasserstoffgemisch in einen Fall, ein Luft-Kohlendioxydge-misch in anderen Fall wird analytisch untersucht. Wasserstoff oder Helium wird gleichmässig durch die poröse Wand in das System eingeblasen. Die Strömung, die Temperatur und die Konzentrationsfelder wurden als Funktionen allein des Wandabstandes angenommen. Nach den Gesetzen der Thermodynamik irreversibler Prozesse sind der Wärme- und Stoffstrom gekoppelt. Der Einfluss dieser Kopplung wird beim Konzentrationsfeld als klein nachgewiesen, beim Temperaturfeld und beim Wärmestrom ist er unter bestimmten Bedingungen zu berücksichtigen; vernachlässigbar ist er für den Wärmeübergangskoeffi-zienten, wenn dieser auf die "adiabate Wandtemperatur" bezogen wird. Allgemein ist die Grösse der Einflüsse wesentlich ausgeprägter für Einblasung von Wasserstoff als von Kohlendioxyd und wirkt im entgegengesetzten Sinn.

Аннотация—Рассматривается установившийся перенос массы и тепла с пористой стенки в нерассеивающую бинарную систему или водородовоздушную смесь в одном случае или смесь воздух-двуокись углерода в другом. Водород или гелий вдуваются в систему через пористую стенку с одинаковой скоростью. Предполагается, что поля течений, температур и концентрации являются функциями расстояния только от стенки. Согласно законам термодинамики необратимых процессов потоки тепла и массы взаимосвязаны. Показано, что воздействие такой взаимосвязи на поле концентрации мало, на поле температур и тепловой поток при определенных условиях довольно ощутимо и пренебре-жимо мало на коэффициент теплообмена, когда он определен через «адиабатическую температуру стенки». Вообще, величины этого воздействия гораздо значительнее при вдуве водорода, чем при вдуве двуокиси углерода, и отличаются по знаку.